HEAT PIPE LINESHAPE

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1. Theoretical description

Figure 1. EMT2 heat pipe layout.

In the EMT2 experiment, we modulate the phase of the heat pipe pump laser, shown in Fig. 1, by driving a LiNbO$_3$ crystal at 11 MHz. Power is transferred to the crystal by arranging it as the capacitive element in an LC tank circuit on resonance with the driving frequency. The laser is linearly polarized, and its electric field is described by$^1$$^2$

\[ E_{\text{pump}} = E_0 e^{i(\omega t + \beta \sin \Omega t)} , \quad (1) \]

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with angular frequency $\omega \approx 447$ THz (671 nm), modulation frequency $\Omega = 11$ MHz, and with the modulation depth $\beta$ determined by the RF drive power, the Q-factor of the LC tank, and the crystal efficiency. For small $\beta$, a Taylor expansion yields

$$
E_{\text{pump}} = E_0 e^{i\omega t} (1 + i\beta \sin \Omega t)
= E_0 e^{i\omega t} \left(1 + \frac{\beta}{2} (e^{i\Omega t} - e^{-i\Omega t})\right)
= E_0 \left(e^{i\omega t} + \frac{\beta}{2} e^{i(\omega + \Omega) t} - \frac{\beta}{2} e^{i(\omega - \Omega) t}\right),
$$

(2)

demonstrating sidebands at the sum and difference frequencies. Inside the heat pipe, the leading-order nonlinear susceptibility of the atomic vapor, $\chi^{(3)}$, mixes any two components of the pump beam with the probe beam $E_{\text{probe}} = E_1 e^{i\omega t}$, which is linearly polarized along the same axis, thus the electric field in the vapor is

$$
E_{\text{vapor}} \propto \chi^{(3)} (E_{\text{pump}} + E_{\text{probe}})^3
= \chi^{(3)} E_0^3 \left(1 + \frac{E_1}{E_0} e^{i\omega t} + \frac{\beta}{2} e^{i(\omega + \Omega) t} - \frac{\beta}{2} e^{i(\omega - \Omega) t}\right)^3.
$$

(3)

Retaining only the terms oscillating at $+\omega$ and not those at $+3\omega$, setting $E_0 = E_1$ for simplicity, and redacting to first order in $\beta$, we have

$$
E_{\text{vapor}} \propto 24\chi_0 e^{i\omega t} + 6\beta \chi_+ e^{i\omega t + i\Omega t} - 6\beta \chi_- e^{i\omega t - i\Omega t},
$$

(4)

where the susceptibility tensor components are functions of the frequency:\footnote{Loudon, R. The Quantum Theory of Light. (Oxford University Press, 1973).}

$$
\chi_0 = \chi^{(3)}[\omega, \omega, \omega],
\chi_+ = \chi^{(3)}[\omega, \omega, \omega + \Omega], \text{ and } \chi_- = \chi^{(3)}[\omega, \omega, \omega - \Omega].
$$

The electric field in the vapor serves as the inhomogeneous source term in the Maxwell wave equation, creating sidebands through the process of four-wave mixing. (The sidebands propagating with the probe beam are resonantly enhanced due to momentum conservation.) On the photodiode, we measure the interference of the probe beam with the sidebands:

$$
V_{\text{diode}} \propto |E_{\text{vapor}} + E_{\text{probe}}|^2
= |24\chi_0 e^{i\omega t} + 6\beta \chi_+ e^{i\omega t + i\Omega t} - 6\beta \chi_- e^{i\omega t - i\Omega t} + e^{i\omega t}|^2,
$$

(5)

where the terms with $e^{i\Omega t}$ are

$$
V_{\text{diode}} \propto -\beta (6\chi_- + 144\chi_0\chi_- - 144\chi_0\chi_+ - 6\chi_+) e^{i\Omega t},
$$

(6)

which demonstrates that the interference of the probe beam carrier with one sideband generates heterodyne beats at frequency $\Omega$. Noting that $\chi^{(3)}$ is small (compared to the linear susceptibility $\chi^{(1)}$) allows the further simplification

$$
V_{\text{diode}} \propto -\beta (6\chi_- - 6\chi_+) e^{i\Omega t}
\equiv -\beta S[\omega, \Omega] e^{i\Omega t}.
$$

(7)

where $S[\omega, \Omega] \equiv 6\chi_- - 6\chi_+$ is the lineshape function which will be discussed shortly. Finally, the diode signal is mixed with the original modulation frequency $\Omega$ to form the error signal:

\footnote{http://www.rp-photonics.com/modulation_depth.html}
\[ V_{\text{error}} = V_{\text{diode}} e^{i\beta \sin \Omega t} \approx - \left( 1 + \frac{\beta}{2} (e^{i\Omega t} - e^{-i\Omega t}) \right)^{\frac{1}{2}} \beta S[\omega, \Omega] e^{i\Omega t} \]
\[ = \left( -e^{i\Omega t} + \frac{\beta}{2} (1 - e^{2i\Omega t}) \right)^{\frac{1}{2}} \beta S[\omega, \Omega], \quad (8) \]
of which the DC component, which survives the 1.9 MHz lowpass filter after the mixer, is
\[ V_{\text{error,DC}} = \frac{\beta^2}{2} S[\omega, \Omega]. \quad (9) \]
This is the heat pipe error signal, and it is proportional to \( \beta^2 \). We observe this signal by sweeping the laser frequency \( \omega \) with the use of an external cavity.

The lineshape function,
\[ S[\omega, \Omega] = 6\chi^{(3)}[\omega, \omega, \omega - \Omega] - 6\chi^{(3)}[\omega, \omega, \omega + \Omega], \quad (10) \]
is effectively maximized when either term forms a cycling transition for the same velocity class of atoms\(^5\). For instance, \( 6\chi^{(3)}[\omega, \omega, \omega + \Omega] \) describes the absorption, emission, and absorption by three photons at \( \omega \), \( \omega \), and \( \omega + \Omega \), in any order. Such a process is simultaneously resonant with the emission of the fourth photon under two conditions, as shown in Fig. 2.

**Figure 2.** Conditions for resonant four-wave mixing, from [5], where the parameter \( \delta \) refers to the modulation frequency \( \Omega \) used in this report. Atomic levels a and b are separated by \( \omega_0 \). Process (a) is resonant for \( \omega = \omega_0 + \Omega/2 \), while process (b) is resonant for \( \omega = \omega_0 - \Omega \).

Therefore, the lineshape bears four features, two from the term \( 6\chi_- \) at center frequencies \( -\Omega \) and \( \Omega/2 \), and two from the term \( 6\chi_+ \) at center frequencies \( -\Omega/2 \) and \( \Omega \), as can be seen as the predominant features in Fig. 3. These pairs of features differ in sign due to the minus sign in Eq. 10. Each of the four features exhibits real and imaginary parts that are 90° out of phase and are related to dispersion and absorption in the vapor, as will be seen in a moment. The width of each individual feature is determined by the natural linewidth (5.9 MHz), which, because it is close to the modulation frequency (11 MHz), results in observable overlap of the features.

A more rigorous approach finds an expression for the shape of each feature by approximately-diagonalizing a set of density matrix equations\(^2 \quad 6 \quad 7 \quad 8\) :
\[ S[\omega, \Omega] = \frac{C}{\sqrt{T^2 + \Omega^2}} \left( [L_{-1} - L_{-1/2} + L_{1/2} - L_1] \cos \Omega t + [D_{-1} - D_{-1/2} - D_{1/2} + D_1] \sin \Omega t \right) \]
\[ \quad \text{where} \]
\[ L_n = \frac{1}{1 + (\frac{\Delta - n\Omega}{\Gamma})^2} \tag{12} \]

\[ D_n = L_n \frac{\Delta - n\Omega}{\Gamma} \tag{13} \]

This expression clarifies that each feature is a combination of a Lorentzian function \( L \) and a dispersion function \( D \). The relative proportion of these functions depends on the phase of the sideband relative to the phase of the driving oscillator, \( \Omega \), at the point where they are mixed together. In our experiment, the relative proportion of these components is not known. However, by adjusting the polarization and/or alignment of the beams, the overall phase can be shifted, which we observe as a change in the lineshape form.

As demonstrated in this report, the lineshape should be a strictly odd function. However, throughout much of 2013, we have observed asymmetry of unknown origin. Additionally, note that the modulation frequency of 11 MHz is close to the natural feature linewidth of 5.9 MHz; thus, if the RF function generator is set to a different frequency (which also decreases the power transferred to the EOM), the lineshape features may be squeezed together or otherwise obscured.

**Figure 3.** Absorption and dispersion patterns resulting from approximately four sidebands (two red, two blue), from [9]. Here \( \omega_m \) represents the modulation frequency \( \Omega \).
2. SOURCES OF LINESHAPE ASYMMETRY AND FREQUENCY DRIFT

When the laser is locked to the heat pipe, its frequency can drift due to any mechanism whose amplitude drifts, where that mechanism introduces a change to the lineshape near $\Delta = 0$ ($\omega = \omega_0$) that is not perfectly odd-symmetric.

In addition, if the lock point is not set to exactly 0 V, then the lock will be sensitive to distortion of the lineshape at $\Delta = 0$, such as due to changes in the slope, a feature at that location, or by stretching the lineshape (i.e., if the sideband frequency drifts).

Whether a certain mechanism can affect the lineshape at $\Delta = 0$ depends on its frequency and the location where it enters the system. For instance, if the mechanism’s characteristic frequency is near the modulation frequency $\Omega$, and it influences the beat signal, $V_{\text{diode}}$, then it can create distortion near $\Delta = 0$.

The following is an incomplete list of possible mechanisms leading to drift:

(1) Higher-order sidebands in the pump lead to new features at the lineshape center$^9$, as shown in Fig. 3. These features may be suppressed by turning down the modulation depth $\beta$, however, this will decrease the sensitivity of the experiment (unless the heat pipe vapor pressure is simultaneously increased). It has been stated$^{10}$ that in our experiment, $\beta \approx 2$; if so, then there are necessarily higher-order sidebands present.

(2) Our modulation frequency is roughly twice the natural linewidth $\Gamma$, which causes the slope of the lineshape at $\Delta = 0$ to be highly sensitive to the exact relative phase of the sideband and the drive$^6$, as is beautifully demonstrated in Fig. 4. Choosing a different frequency, or introducing a controllable phase shift between the pump and probe beams, could ameliorate this sensitivity.

Figure 4. Sensitivity of the lineshape to the modulation frequency, from [6].


(3) If there is 11 MHz noise picked up at the photodiode, it will change the lock point. This problem would be exacerbated if the lineshape near $\Delta = 0$ is nearly flat, as discussed in the previous item.

(4) The EOM crystal is birefringent\textsuperscript{11}, so drifting of the modulation RF power inside the crystal will change the relative phase of the pump sidebands, and thereby change the amount of power in the probe sidebands. In addition, it will cause the polarization of the pump beam to drift\textsuperscript{10}, which can cause some of the pump beam to enter the photodiode at the beamsplitting cube. (Changes in RF power could be caused by mechanical resonances whose amplitude drifts in time, thereby robbing energy from the driving frequency, or via drifting in the impedance of the LC tank circuit. In addition, the RF generator or amplifier power could drift. Previous documentation\textsuperscript{10} suggests that the spectral impurity of the RF drive enhances the possibility of absorption by a mechanical resonance.)

(5) According to a previous discussion of this apparatus\textsuperscript{10}, the lineshape is riding on the shoulder of the overall lineshape of the probe scan. This refers to the very broad shape observed in the left pane of Fig. 5. The figure demonstrates that this broad profile is dominated by dispersion, which has odd symmetry. If the probe power changes, this would change the magnitude of the underlying absorption component, which has even symmetry and would therefore produce an overall vertical drift of the lineshape. The same effect would be observed if the linear susceptibility $\chi^{(1)}$ changes in time, such as if the magnetic field drifts.

**Figure 5.** Lineshape in our experiment, from [10].

(6) The counter-propagating beams create a standing-wave pattern in the population of excited atoms, which can function as a Bragg grating\textsuperscript{9}. Reflections of various sidebands off this grating introduce new sidebands into the opposite beam. For instance, interference between a probe sideband and the pump carrier would produce a moving grating; reflection of the probe carrier off this grating would produce a sideband in the pump beam. If the lasers point at an angle which drifts in time, the velocity of the grating will drift, introducing a drifting slope in the lineshape at $\Delta = 0$. In addition, if the laser overlap over a large distance, any effects of the grating will be larger and therefore more sensitive to other changes in the experiment.

(7) If the beams are converging or diverging through the atomic vapor, the lineshape can shift due to the curvature of the wavefronts\textsuperscript{8}, shown in Fig. 8. The greatest wavefront curvature of a Gaussian beam occurs at the Rayleigh length. If the focal positions of the beams are

\textsuperscript{11} http://www.thorlabs.com/newgrouppage9.cfm?objectgroup_id=2729
drifting in time, such as due to an aging laser diode or heating of the optical components, then the lineshape will be shifted, losing its odd symmetry.

**Figure 6.** Lineshape due to curved wavefronts, from [8].